

Supplementary Materials

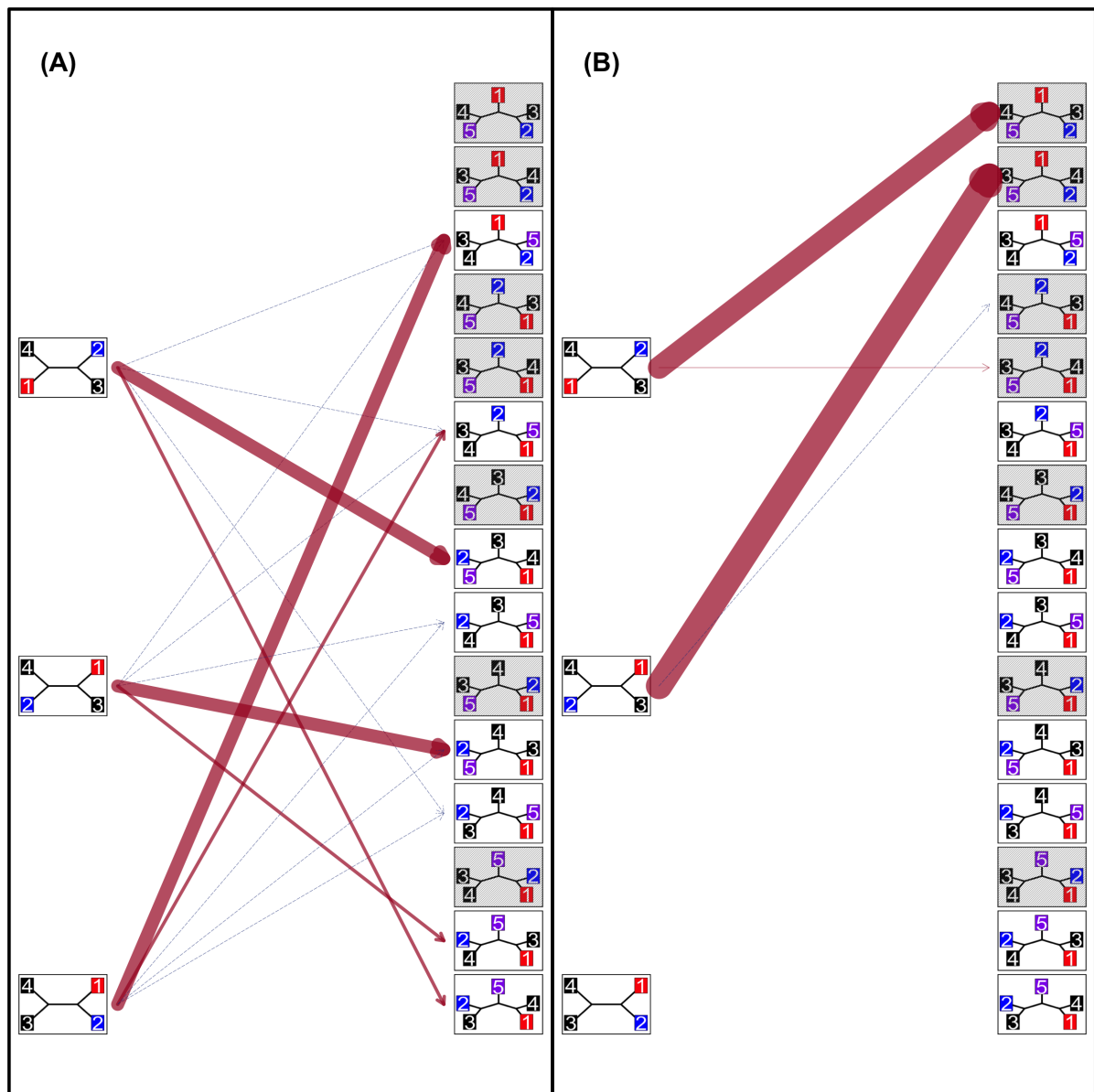


Figure S1: Topology analysis of Property 3 at $\alpha = 0.01$ under a random matrix model with admixture. (A) Simulations in which Property 3 holds. (B) Simulations in which Property 3 fails. Nodes on each graph represent all possible unrooted binary labeled topologies for $n = 4$ (left column) and $n = 5$ (right column). The 4-taxon source NJ tree $\mathcal{T}_S^{(4)}$ is constructed from $\mathbf{S}^{(4)}$, and the 5-taxon admixed NJ tree $\mathcal{T}_D^{(5)}$ is inferred from the associated $\mathbf{D}^{(5)}$. The NJ tree topologies are classified and assigned to nodes. A directed edge from $\mathcal{T}_S^{(4)}$ to $\mathcal{T}_D^{(5)}$ corresponds to a given ordered pair of distance matrices $(\mathbf{S}^{(4)}, \mathbf{D}^{(5)})$ from which the connected trees are constructed. The edge weight is proportional to the frequency of a connected tree pair's occurrences, with the exception that for those with nonzero frequencies less than 1%, the edges are kept at 1% weight and are indicated with blue dotted lines. Shaded nodes represent all possible topologies with $n = 5$ that violate Property 3 given that t_1 and t_2 are source taxa and t_5 is the admixed taxon. Because only 4 of 7 shaded topologies are attainable (Corollary 2), only those 4 shaded topologies have incoming edges in panel (B). The node $((t_1, t_2), (t_3, t_4))$ does not have an outgoing edge in panel (B) because Property 3 holds when $\mathcal{T}_S^{(4)} = ((t_1, t_2), (t_3, t_4))$ (Proposition 3). The simulations shown correspond to those in Figure 5 and Table 4.

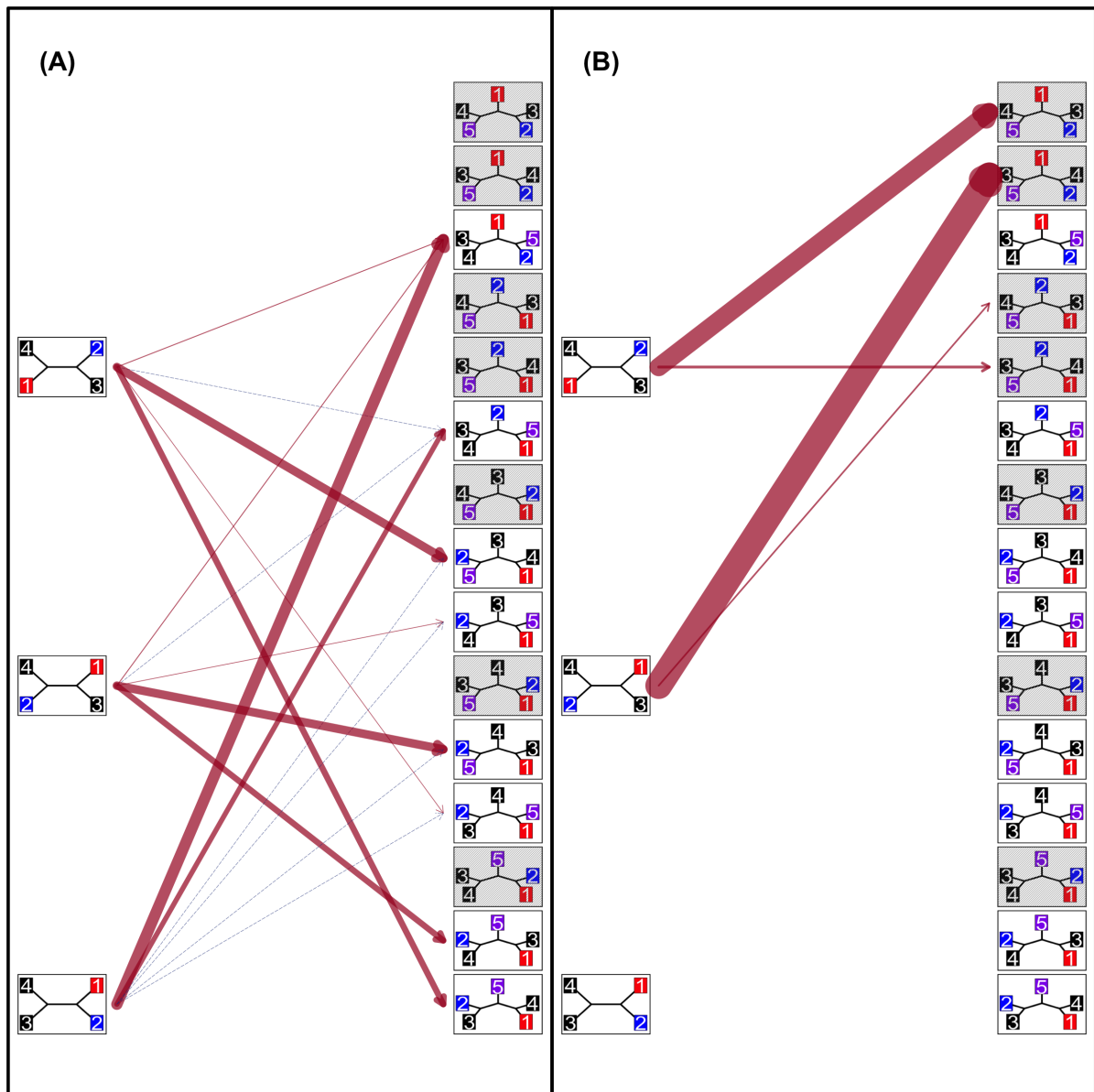


Figure S2: Topology analysis of Property 3 at $\alpha = 0.25$ under a random matrix model with admixture. (A) Simulations in which Property 3 holds. (B) Simulations in which Property 3 fails. The plots are constructed using the same procedure as in Figure S1.

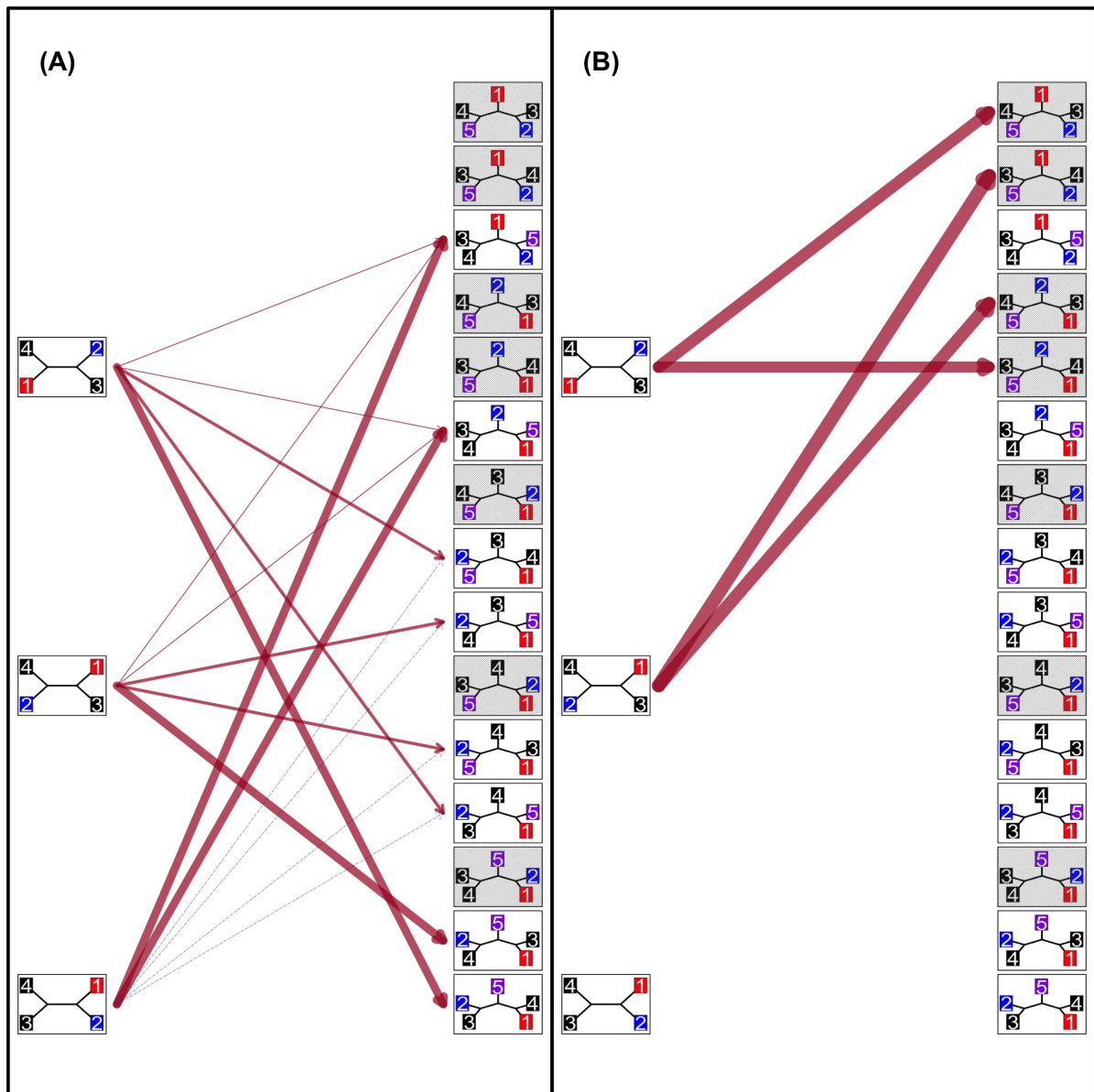


Figure S3: Topology analysis of Property 3 at $\alpha = 0.5$ under a random matrix model with admixture. (A) Simulations in which Property 3 holds. (B) Simulations in which Property 3 fails. The plots are constructed using the same procedure as in Figure S1.